POWER SYSTEMS ENGINEERING

PER UNIT SYSTEMS

TEN TOTAL PROBLEMS SOLVED WITH DETAILED EXPLANATION

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FROM THE AUTHOR

I CAME ACROSS PER UNIT SYSTEMS WHEN I WAS STUDYING EE FOR MY BACHELORS. I CAME ACROSS IT AGAIN DURING MY MASTERS PROGRAM. AND NOW TEN YEARS LATER - DECEMBER 2018, I STILL DEAL WITH IT AS A SUBSTATION ELECTRICAL ENGINEER AT A LOCAL ELECTRIC UTILITY.

DO NOT TAKE THIS TOPIC LIGHTLY IF YOU PLAN ON FOLLOWING MY PATH INTO THE POWER INDUSTRY.

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ALEEN

TABLE OF CONTENT

01 WHY USE PER UNIT SYSTEMS

Fundamental to any power system analysis is the know-how of per unit systems. This metric is widely used to describe voltages, currents, and impedances in a power system. This book, supplemented by plenty of examples, will explain how to calculate these parameters for any component anywhere in the power system. Let's begin with why use per unit systems.

1. Transformers and per unit systems

The reason the per unit system is employed is because it simplifies calculation of voltages, currents, and impedances of a power system network. Imagine a 3-phase system with a generator, a transformer, a T-line, another transformer, and a load. Every time you cross the transformers, the currents and voltages change. The impedance of the transformer referred to the primary side is different from the impedance of the same transformer referred to the secondary side. It is imperative to use the appropriate impedance based on which side of the transformer you are on, which if not accounted for, introduces serious calculation errors.

Using actual values is fine if not for the transformers. They complicate matters. Generators and motors, like transformers, have different impedance when referred to either stator or rotor winding.

Deriving a per unit impedance values for these components enables easy calculation. This is because the per unit value remains the same whether you are on the primary side or the secondary side of the equipment.

2. The $\sqrt{3}$ factor

The second factor in choosing per unit systems is the avoidance of $\sqrt{3}$ factor in the calculations. If you have ever tried calculating either the sending end voltage or the receiving end voltage in a system, you would know how important it is to factor in the $\sqrt{3}$ multiplier. This is because the voltages are always specified in phase to phase (L-L) values while the voltage drop across the impedances are calculated on per phase (L-N) basis. You will have to use $\sqrt{3}$ factor extensively to switch between L-L and L-N values.

When using the per unit system, a 3-phase balanced network is reduced to a 1-phase equivalent where the $\sqrt{3}$ isn't even in the picture.

3. Intuition enabler

The third factor is the meaning expressed by the per unit values. Look at the figure below.

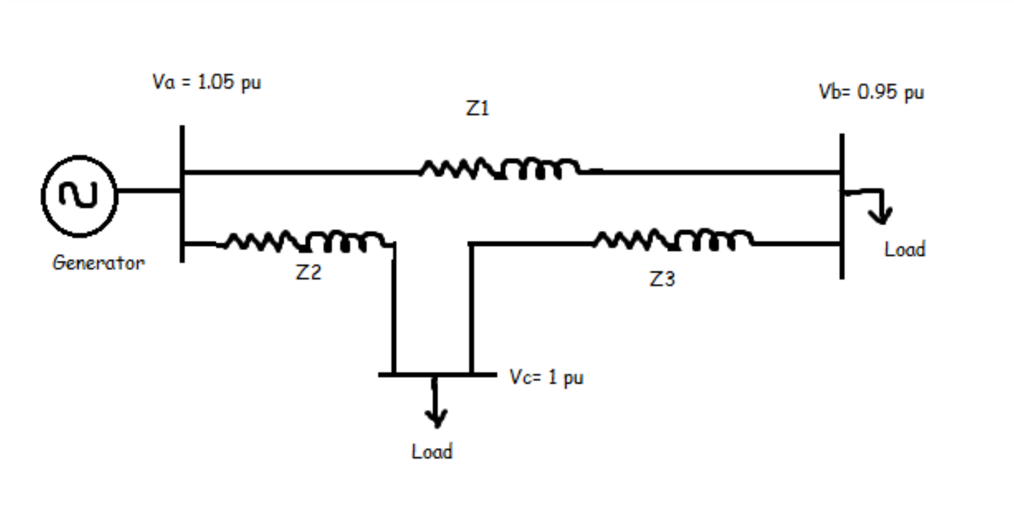


Figure 1: Voltage Per Unit

What does it tell about the system voltage profile? The generator is generating 5% above rated voltage. The load at bus C has no drop in voltage (1 pu) while the load at bus B sees 5% drop in voltage. Essentially, the voltage drops as you move away from the generator and closer to the load. It is easier to visualize this using per units than using actual values since the per unit values are normalized.

Also, someone with experience knows what the per unit values, especially impedance, of certain transformers or other devices in the system could be. It is easier for them to analyze the power network by making reasonable assumptions.

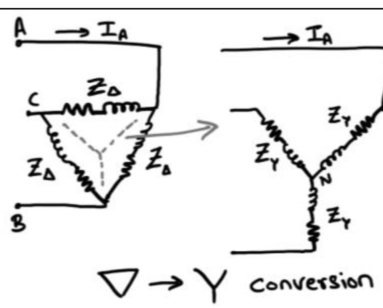
Other factors for employing per unit system include

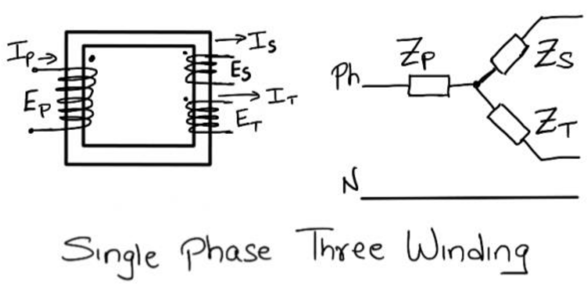
01 WHY USE PER UNIT SYSTEMS

- enables the use of symmetrical components.
- widely used by manufacturers to specify their equipment – making it easy for direct analysis.

At this point it is sufficing to say the per unit system is indispensable in analyzing the power system

02 EQUATIONS

No.	Equation	Comments
1	$Z_{pu_{new}} = Z_{pu_{old}} \times \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2$	Normally per unit impedance of equipment is furnished on its nameplate S and V rating. Use this equation to calculate new impedance when either one or both ratings need to change to another base.
2	$Z_{pu} = Z_{\Omega} \times \frac{MVA_{base}}{kV_{base}^2}$ Where, $Z_{base} = \frac{kV_{base}^2}{MVA_{base}}$	Useful in calculating transmission line per unit impedance. Plug in line impedance in ohms and the desired S_{base} and V_{base}.
3	$V_{Secbase} = V_{Pribase} \times \frac{N_s}{N_p}$ Since voltage developed in transformer is proportional to turns ratio $V_{Secbase} = V_{Pribase} \times \frac{V_s}{V_p}$	Calculate the new V_{base} on the secondary side of transformer provided a V_{base} is selected on the primary side or vice versa. See solved problems 1 or 2.
4	$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$	Formula for per unit quantity. The same applies for calculating V, I, or S in per unit.
5	$Z_{pri\_referred} = \left(\frac{N_1}{N_2}\right)^2 \times Z_{Sec\_impedance}$	Use this formula to transfer the impedance on the secondary side of transformer to the primary side.
6	 $Z_Y = \frac{Z_\Delta}{3}$	A balanced Δ load can be converted to a balanced Y load using this equation. Also, the line current entering Y will be equal to that entering delta. The phase angles are not affected and remain the same. A balanced Y can be converted into Δ using this same expression. See solved problem 5.
7	$V_{AB} = \sqrt{3}V_{an}\angle + 30°$ $V_{BC} = \sqrt{3}V_{bn}\angle + 30°$	In a balanced Y connected load, line-to-line voltages are $\sqrt{3}$ times line-to-neutral and lead by 30°.

	$V_{CA} = \sqrt{3}V_{cn}\angle + 30°$ complex power: $S_{3\emptyset} = \sqrt{3}V_{LL}I_L^* = P_{3\emptyset} + jQ_{3\emptyset} = \sqrt{3}V_{LL}I_L \cos\emptyset + j\sqrt{3}V_{LL}I_L \sin\emptyset$	In a balanced Δ connected load, line currents into the load are $\sqrt{3}$ times the Δ-load currents and lag by $30°$. $\emptyset$ is the angle between voltage and current. In purely resistive network, current is in phase with voltage. Therefore $\emptyset$=0. In inductive network, current lags voltage. In capacitive network, current leads voltage. See solved problem 4.
8	Equation to calculate impedance of a single phase three-winding transformer  $Z_P = \frac{1}{2}(Z_{PS} + Z_{PT} - Z_{ST})$ $Z_S = \frac{1}{2}(Z_{PS} + Z_{ST} - Z_{PT})$ $Z_T = \frac{1}{2}(Z_{PT} + Z_{ST} - Z_{PS})$	Here, Z_{PS}= per unit leakage impedance measured from primary winding, with secondary shorted and tertiary open Z_{PT}= per unit leakage impedance measured from primary winding, with tertiary shorted and secondary open Z_{ST}= per unit leakage impedance measured from secondary winding, with tertiary shorted and primary open These impedances (Z_{PS}, Z_{PT} & Z_{ST}) can be obtained from the transformer manufacturer based on their short-circuit tests pursuant to IEEE standards.

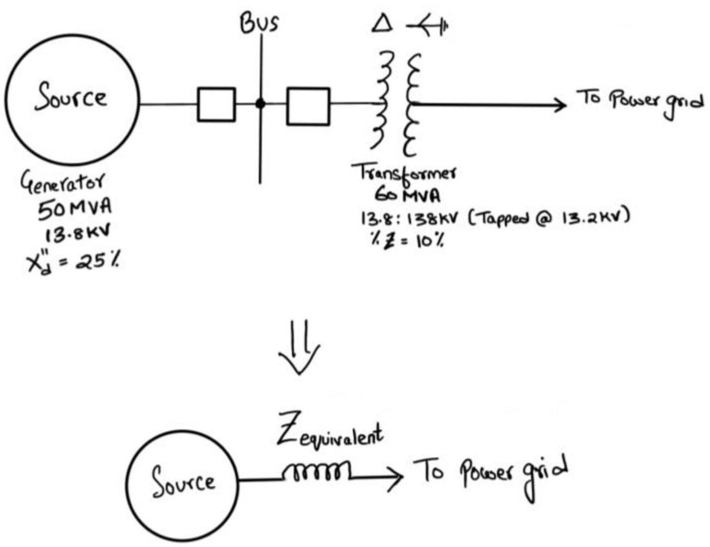
03 PROBLEMS SOLVED

For the problems solved below, few general assumptions need to be made about the power system. These may be reiterated in the question.

1. Line shunt capacitance of a transmission line is neglected.
2. Magnetizing reactance of transformer is ignored. Only leakage reactance is used in the per unit impedance drawings.

Question 1:

A generator and its step-up transformer are shown. The (direct-axis) sub-transient reactance of generator (in per unit) and transformer impedance (in per unit) are provided. Both per unit impedances are based on their nameplate rating. Calculate equivalent impedance of this system using S_{base} =100MVA and V_{base}=132kV.



Solution:

The challenge with this problem is both impedances are based on equipment nameplate. Also, the V_{base} specified is different from that of the transformer's voltage rating. The goal therefore is to convert the given impedances to new ones based on new S_{base} and V_{base} ratings.

With the transformer bank operating on its 13.2kV tap, the low-side base voltage corresponding to the 132kV high-side base (using equation 3) is:

$$\frac{kV_{LV}}{132} = \frac{13.2}{138} \ or \ kV_{LVbase} = 12.626kV$$

12.62kV is the new base voltage in the 13.8kV portion of the system.

The new pu impedance based on equation (7) from chapter 2 is

$$X_d^{"} = 25\% \times \frac{100 \times 13.8^2}{50 \times 12.626^2} = 60\% \; or \; 0.6pu$$

$$\text{on 100MVA 12.626kV base}$$

$$\text{or on 100MVA 132kV base}$$

Similarly, the transformer impedance on the new base is

$$Z_T = 10\% \times \frac{100 \times 13.2^2}{60 \times 12.626^2} = 10\% \times \frac{100 \times 138^2}{60 \times 132^2}$$

$$= 18.22\% \; or \; 0.182pu$$

$$\text{on 100MVA 12.626kV base}$$

$$\text{or on 100MVA 132kV base}$$

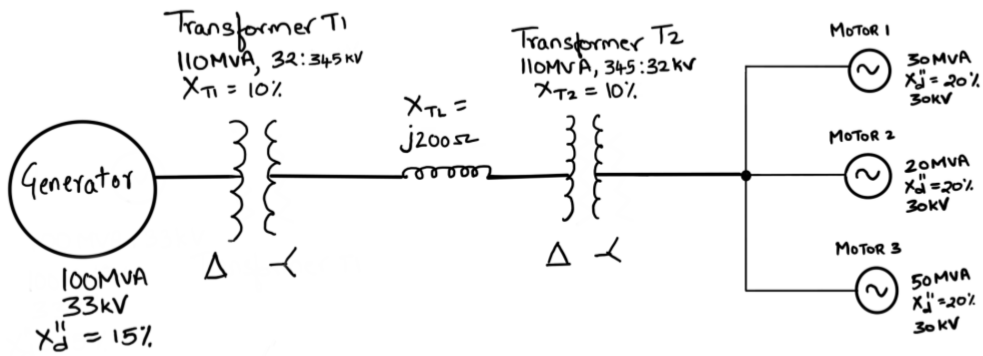
The equivalent impedance is

$$\text{Zequivalent} = 0.6\text{pu} + 0.182\text{pu} = \mathbf{0.782pu} \; on \; a \; 100MVA \; 132kV \; base$$

03 PROBLEMS SOLVED

Question 2:

For the power system shown below, draw the per unit impedance circuit. Select the generator S and V ratings as the base quantities.



Solution:

With base values as 100MVA and 33kV in the generator circuit, the p.u. reactance of generator will be 15% or **0.15pu**.

Let's determine the base voltages for the transmission line and for the motors.

The base value of voltage in the line (using equation 3) will be

$$33 \times \frac{345}{32} = 355.78kV$$

In the motor circuit,

$$355.78 \times \frac{32}{345} = 33kV$$

The reactance of transformers T1 and T2 is 8% corresponding to 110MVA, 32kV. Therefore, corresponding to new S_{base} = 100MVA and V_{base} = 33kV, the p.u. reactance using equation (7) will be

$$0.08 \times \frac{100}{110} \times \left(\frac{32}{33}\right)^2 = \mathbf{0.06838pu}$$

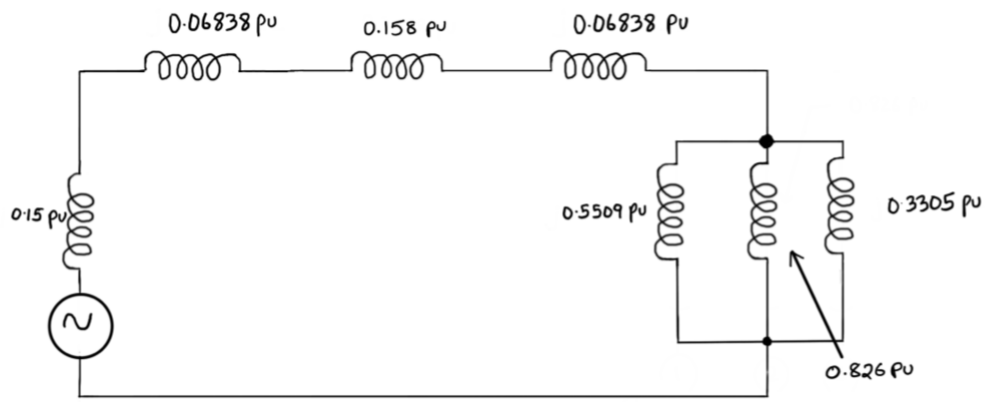
The p.u. impedance of line using equation (2) = $\frac{200 \times 100}{(355.78)^2} = \mathbf{0.158pu}$

The p.u. reactance of motor 1 = $0.2 \times \frac{100}{30} \times \left(\frac{30}{33}\right)^2 = \mathbf{0.5509pu}$

Motor 2 = $0.2 \times \frac{100}{20} \times \left(\frac{30}{33}\right)^2 = \mathbf{0.826pu}$

Motor 3 = $0.2 \times \frac{100}{50} \times \left(\frac{30}{33}\right)^2 = \mathbf{0.3305pu}$

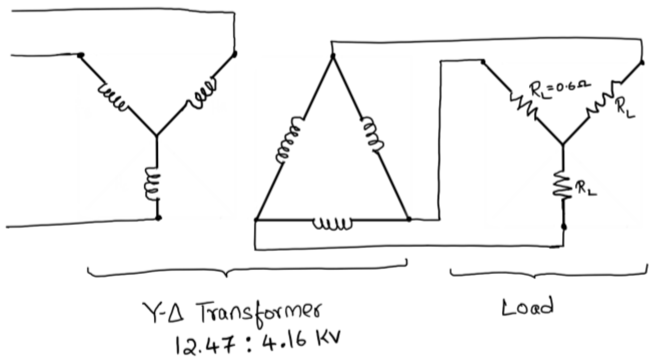
The reactance diagram for the system is shown in figure:



03 PROBLEMS SOLVED

Question 3:

Three single phase, two-winding transformers, each rated 25 MVA, 7.2/4.16kV, are connected to form a three-phase Y-Δ bank with a balanced Y connected resistive load of 0.6Ω per phase on the low voltage side. By choosing a base of 75MVA (three phase) and 12.47kV(line-to-line) for the high voltage side of the transformer bank, specify the base quantities for the low-voltage side. Then determine the load resistance of R_L in ohms referred to the high-voltage side and the per-unit value of this load resistance on the chosen base.



Solution:

The L-L voltage on the high-side transformer is $\sqrt{3}\,(7.2) = 12.47kV\ (V_{L-L} = \sqrt{3}V_{L-N})$

The 3-phase transformer rating: 75 MVA ($S_{3\emptyset} = 3 \times S_{1\emptyset}$), 12.47Y/4.16$\Delta$ kV.

Base impedance for the low-voltage side (using equation 2) is $\frac{(4.16)^2}{75} = \mathbf{0.231\Omega}$

On the low-voltage side (using equation 4), $R_L = \frac{0.6}{0.231} = 2.6\ pu$

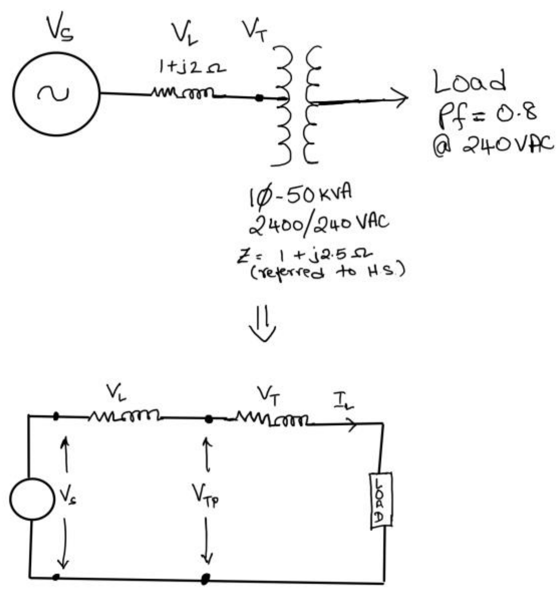
Base impedance on high-voltage side is $\frac{(12.47)^2}{75} = 2.07\Omega$

The resistance referred to high voltage side (using equation 5) is $0.6\left(\frac{12.47}{4.16}\right)^2 = \mathbf{5.39\Omega}$

Or $R_L = \frac{5.39}{2.07} = \boldsymbol{2.6\ pu}$ on 75MVA and 12.47kV base

Question 4:

A single-phase 50kVA, 2400/240volt, 60Hz distribution transformer is used as a step-down transformer at the load end of a 2400-volt feeder whose series impedance is (1.0+j2.0) ohms. The equivalent series impedance of the transformer (1.0 +j2.5) ohms referred to the high-voltage (primary) side. The transformer is delivering 1 per unit current at 0.8 power factor lagging and at rated secondary voltage. Neglecting the transformer exciting current, determine (a) the voltage at the transformer primary terminals, (b) the voltage at the sending end of the feeder, and (c) the real and reactive power delivered to the sending end of the feeder. (d) Using the transformer ratings as base quantities, work the problem in per-unit.



Solution:

On secondary side of transformer, assume $V_{load} = 1.0 \angle 0°$pu

At 0.8 power factor, the current lags behind voltage by $\cos^{-1} 0.8 = \angle -36.87°$

Therefore $\bar{I}_{L\_pu} = 1.0 \angle -36.87°$ pu

On primary side of transformer,

S_{base}=50KVA

V_{base}=2400V

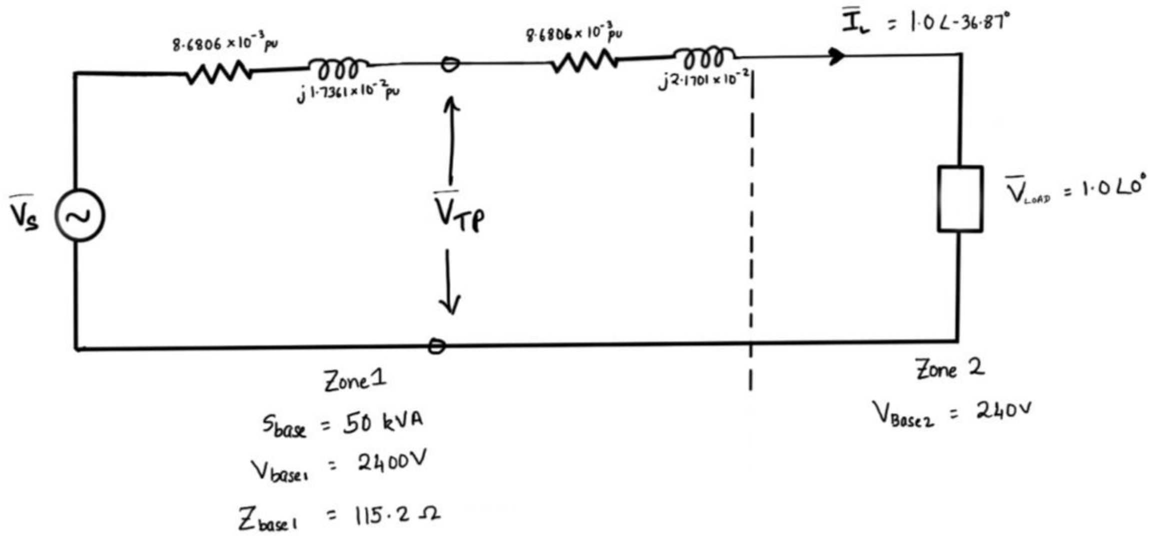
Therefore (using equation 2) $Z_{base\_pri} = \frac{2.4^2}{0.05}$= 115.2 Ω

03 PROBLEMS SOLVED

Transformer impedance in pu = $\frac{1+j2.5}{115.2} = 8.6806 \times 10^{-3} + j2.1701 \times 10^{-2}$ pu

Line impedance in pu=$\frac{1+j2}{115.2} = 8.68.06 \times 10^{-3} + j1.736 \times 10^{-2}$ pu

The equivalent network:



(a) $\overline{V}_{TP} = \overline{V}_{LOAD} + \overline{V}_T = 1.0\ \angle 0° + (8.6803\ \times\ 10^{-3} + j2.1701\ \times 10^{-2})(1.0 \angle -36.87°)$
$= 1.0 + 0.023373\ \angle 31.33° = 1.01997 + j0.012157$
$= 1.020 \angle 0.683°\ pu$

$\overline{V}_{\text{T\_act}} = \overline{V}_{pu}\text{V}_{base}\ =\ (1.020 \angle 0.683°)\ (2400)\ =\ \mathbf{2448 \angle 0.683°\ volts}$

(b) $\overline{V}_{TP} = \overline{V}_{LOAD} + (\overline{Z}_T + Z_L)\overline{I}_L\ =\ \overline{V}_{S\_pu} = 1.0\ \angle 0° + (1.7631 \times 10^{-2} + j\ 3.9063 \times 10^{-2})(1.0 \angle -36.87°)$
$= 1.0 + 0.042747\ \angle 29.168°$
$= 1.03733 + j\ 0.020833$
$\overline{V}_{S\_pu} = 1.0375\ \ \angle 1.1505°\ pu$

$\overline{V}_{\text{S\_act}} = \overline{V}_{S\_pu} \times \text{V}_{base}\ =\ (1.0375 \angle 1.1505°)\ (2400)\ =\ \mathbf{2490 \angle 1.1505°}$ **volts**

(c) $P_{S\_pu} + iQ_{S\_pu}\ =\ \overline{V}_{S\_pu}\ \times \overline{I}^*_{L\_pu} = (1.0375 \angle 1.1505°)(1.0 \angle +36.87°)$ (using equation 7)

03 PROBLEMS SOLVED

In a power system, for a load, when P and Q are positive it means the load absorbs both active (P) and reactive (Q) power. If either P or Q is negative, it means the load is *producing* P or Q.

For a generator, the convention is exactly the opposite. P and Q are positive when generator is producing and injecting P or Q into the network. If P is negative then it is absorbing active power. If Q is negative then it is absorbing reactive power.

In this problem both P and Q are positive for the generator, shown below.

$P_{S\_pu} + iQ_{S\_pu} = 1.0375 \angle 38.02° = 0.8173 + \mathrm{j}0.6390$ per unit

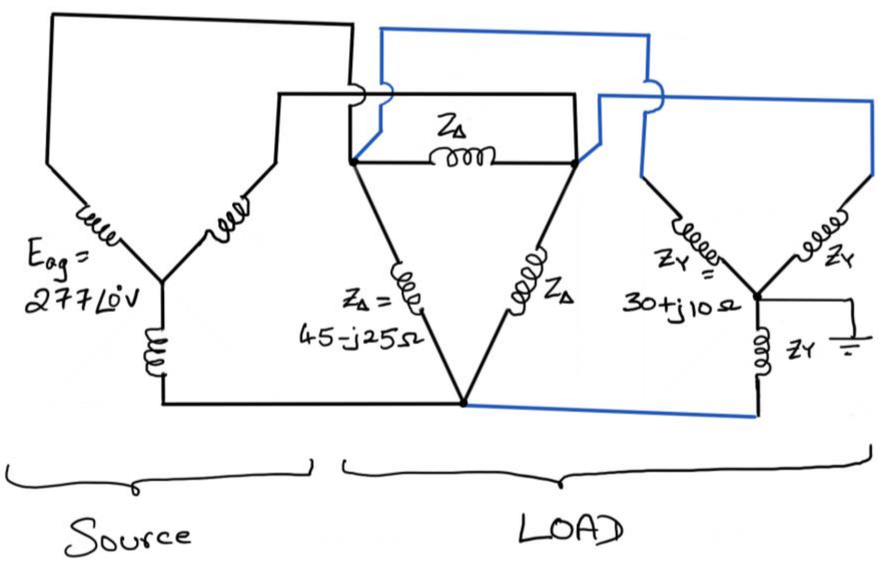
In actual units, using equation 4,

$P_s = (0.8173)(50) = \mathbf{40.87}\boldsymbol{kW}$ delivered

$Q_s = (0.6390)(50) = \mathbf{31.95}\boldsymbol{kVars}$ delivered

Question 5:

A balanced Y-connected voltage source with $E_{ag} = 277\angle 0°$volts is applied to a balanced Y-load in parallel with a balanced-Δ load, where $Z_Y = 30 + j10\ and\ Z_\Delta = 45 - j25ohms.$ The Y load is solidly grounded. Using base values of $S_{base1\phi} = 5kVA$ and $V_{baseLN} = 277$ volts, calculate the source current I_a in per-unit and in amperes.



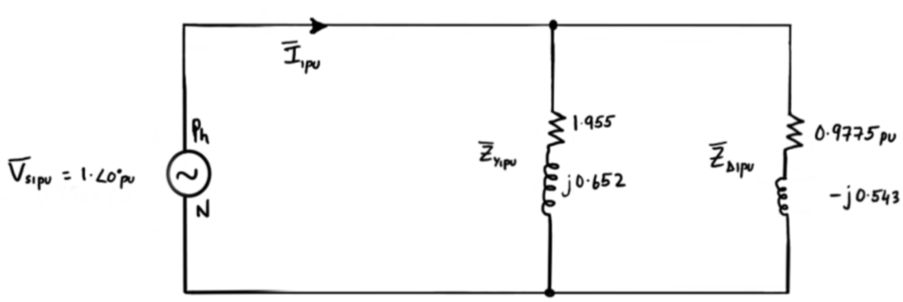
Solution:

$Z_{base} = \frac{277^2}{5000} = 15.346\Omega$ (using equation 2); $I_{base} = \frac{5000}{277}$ = 18.05A

$\overline{z}_{Y1\,pu} = \frac{30+j10}{15.346}$ = 1.955 + j0.652 = 2.061$\angle 18.43°$pu

$\overline{z}_{\Delta_1 pu} = \frac{45-j25}{3\times 15.346} = 0.9775 - j0.543$ = 1.118$\angle - 29.05°$pu

(Note that the delta load was changed to equivalent Y in the above calculation using equation 6)



$$\bar{I}_{1\,pu} = \frac{\bar{V}_{S1\,pu}}{\bar{Z}_{Y1\,pu} \,\|\, \bar{Z}_{\Delta_1 pu}}$$

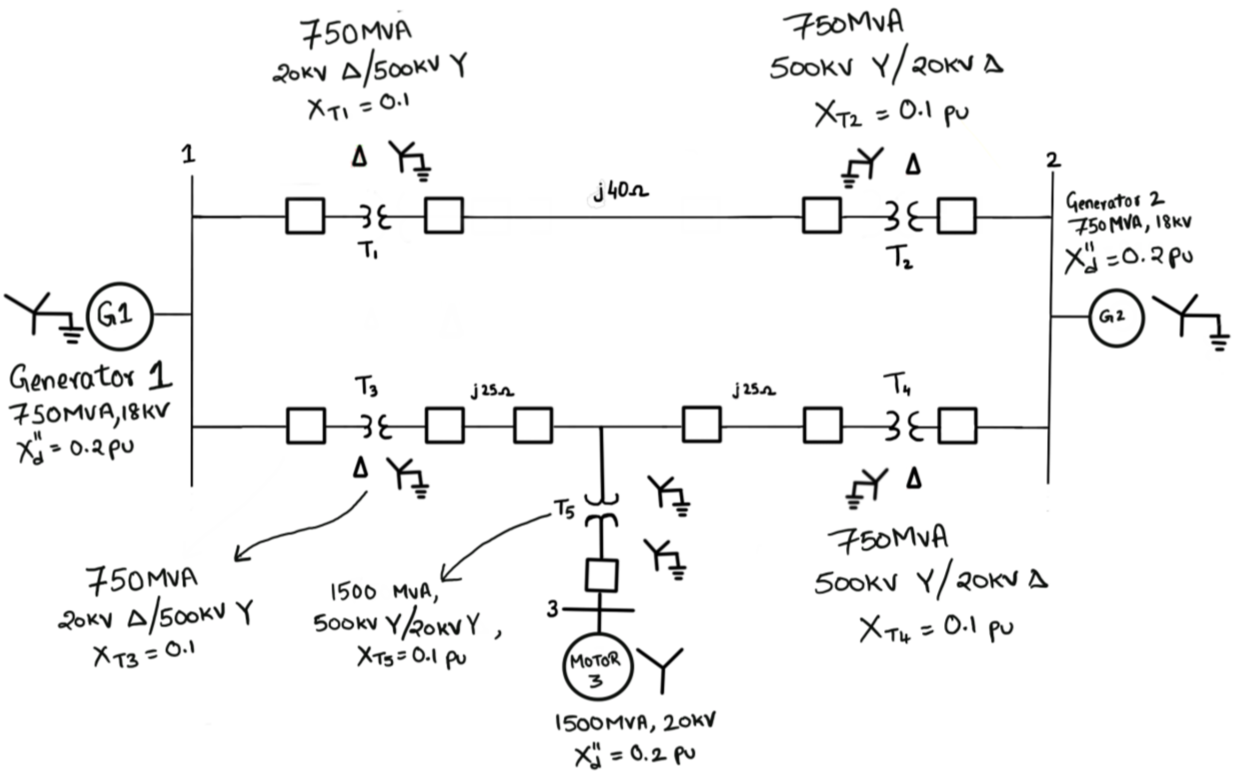
$$= \frac{1\angle 0°}{\frac{(2.061\angle 18.43°)(\,1.118\angle -29.05°)}{(1.955 + j0.652) + (0.9775 - j0.543)}}$$

$$= 1.274\angle 12.74°\ pu$$

$$\bar{I}_{1\_actual} = \bar{I}_{1\,pu}\ I_{base} = (1.274\angle 12.74°)(18.05) = 22.99\angle \mathbf{12.74}°\text{Amps}$$

Question 6:

Consider the single-line diagram of the power system shown in figure.



Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the equivalent reactance diagram. Use a base of 100MVA and 500kV for the 40-ohm line. Determine the per-unit reactances.

Solution:

$S_{base} = 100\ MVA$

$V_{baseH} = 500kV$ in transmission line zones

$V_{baseX} = 500 \times \frac{20}{500} = 20kV$ in motor/generator zones (using equation 3)

$X_{g1}'' = X_{g2}'' = (0.2)\left(\frac{18}{20}\right)^2 \left(\frac{100}{750}\right) = \mathbf{0.0216}$ **pu** (using equation 1)

$X_{m3}^{11} = (0.2)\left(\frac{100}{1500}\right)\left(\frac{20}{20}\right) = \mathbf{0.01333}$ **pu** (using equation 1)

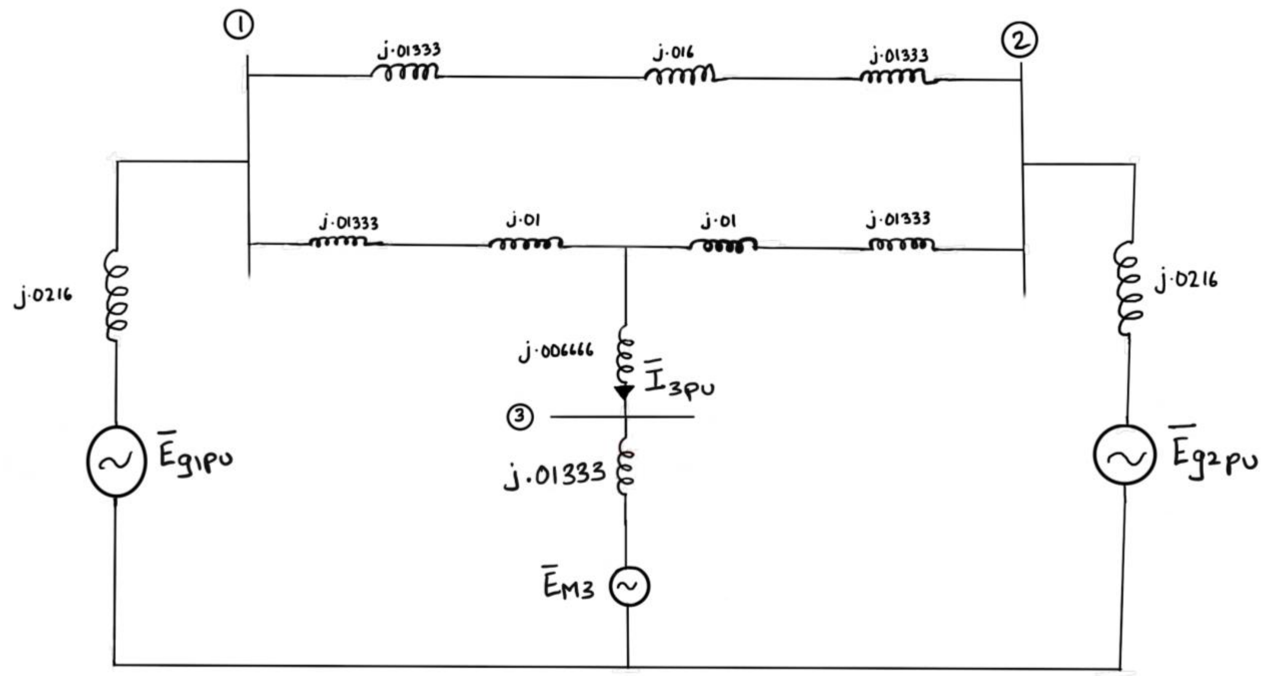
$X_{T1} = X_{T2} = X_{T3} = X_{T4} = (0.10)\left(\frac{100}{750}\right) = \mathbf{0.01333}$ **pu** (using equation 1)

$X_{T5} = (0.10)\left(\frac{100}{1500}\right) = \mathbf{0.00666}$ pu

$Z_{baseH} = \frac{(500)^2}{100} = 2500.\,\Omega$

$X_{Line40} = \frac{40}{2500} = \mathbf{0.016}$ pu

$X_{Line25} = \frac{25}{2500} = \mathbf{0.01}$ pu



03 PROBLEMS SOLVED

Question 7:

For the power system problem above, the synchronous motor absorbs 1200MW at 0.8 power factor leading with the bus 3 voltage at 18kV. Determine the bus 1 and bus 2 voltages in kV. Assume that generators 1 and 2 deliver equal real powers and equal reactive powers. Also assume a balanced three-phase system with positive-sequence sources.

Solution:

$\overline{V}_{3pu} = \frac{18}{20}\angle 0° = 0.9\angle 0°$ pu

$\overline{I}_3 = \frac{1200\times 10^3}{(\sqrt{3})(18)(0.8)}\angle \cos^{-1}(.8) = 48.11\angle +36.87°\ kA$

Note the positive angle for the current. This sign was chosen because the motor current was indicated as leading. In other words, the motor is supplying VARs.

$$I_{baseX} = \frac{S_{base}}{\sqrt{3}\times V_{base}} = \frac{100\times 10^3}{\sqrt{3}\,(20)} = 2.887\ kA$$

$$\overline{I}_{3pu} = \frac{48.11\angle 36.87°}{2.887} = 16.67\angle 36.87° \text{pu}$$

In this problem it is assumed Generator 1 and 2 deliver the same P and Q. This means the current entering Bus 3 from Generator 1 and from Generator 2 is equal. In other words, Current I_3 can be split in half, coming from bus 1 and bus 2.

$$\overline{V}_{1pu} = \overline{V}_{3pu} + \overline{I}_{3pu}\left(jX_{T5pu}\right) + \frac{1}{2}\overline{I}_{3pu}\left(jX_{Line25pu} + jX_{T3pu}\right)$$

$$\overline{V}_{1pu} = \overline{V}_{2pu} = \overline{V}_{3pu} + \overline{I}_{3pu}\, j\left(X_{T5pu} + \frac{X_{Line25pu} + X_{T3pu}}{2}\right)$$

Similarly

$$\overline{V}_{2pu} = \overline{V}_{3pu} + \overline{I}_{3pu}\, j\left(X_{T5pu} + \frac{X_{Line25pu} + X_{T4pu}}{2}\right)$$

Because $X_{T3PU} = X_{T4PU}$

$$\overline{V}_{1pu} = \overline{V}_{2pu} = 0.9\angle 0° + 16.67\angle 36.87°\,(j)\left(.00666 + \frac{.01 + .01333}{2}\right)$$

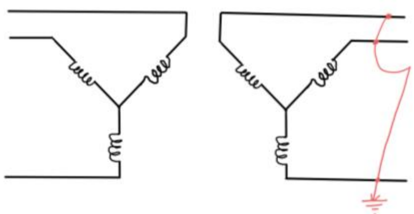
$= .9 + 0.30555\angle 126.87° = 0.7167 + j0.2444 = 0.7572\angle 18.83°$ pu

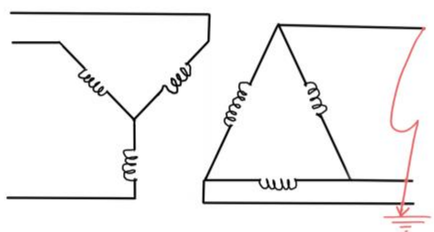
$V_{1\_actual} = V_{2\_actual} = (0.7572)(20) = \mathbf{15.14\ kV}$ (using equation 4)

03 PROBLEMS SOLVED

Question 8:

Three-single phase transformers, each rated at 10MVA 66.4/12.5kV, 60Hz, with an equivalent series reactance of 0.12 per unit divided equally between primary and secondary, are connected in a three-phase bank. The high-voltage windings are Y connected and their terminals are directly connected to a 115-kV three-phase bus. The secondary terminals are all shorted together. Find the currents entering the high-voltage terminals if the low-voltage windings are (a) Y connected, (b) Δ connected.



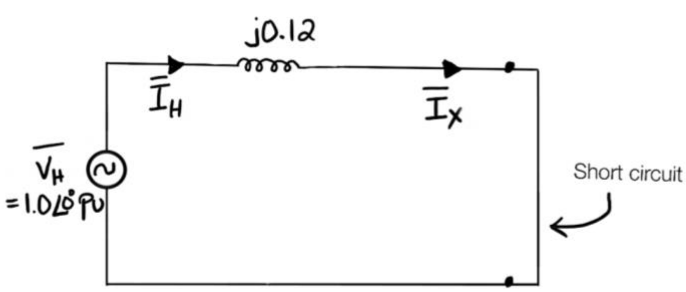


Solution:

$S_{base3\phi} = 30MVA$

$V_{HL-L} = \sqrt{3} \times 66.4 = 115kV$ (using equation 7)

$I_{Hbase} = I_{Xbase} = \frac{1.0\angle 0°}{j0.12} = 8.333\angle -90°\ pu$ (fault current in per unit)



(a) $I_{Hact} = \frac{30\times10^3}{\sqrt{3}\times115} = 0.1506kA$; $V_{XbaseL-L} = 12.5\sqrt{3} = 21.65kV$

$$I_{Xact} = \frac{30}{21.65\sqrt{3}} = 0.8kA$$

$|I_H| = (8.333)(0.1506) = \mathbf{1.255kA}$ (using equation 4)

$|I_X| = (8.333)(0.8) = 6.666kA$ (current on the low side too is calculated and presented here)

(b) $I_{Hact} = 0.1506kA$; $V_{Xact} = 12.5kV$ (In a Delta $V_{L\text{-}L} = V_{L\text{-}N}$)

$$I_{Xact} = \frac{30 \times 10^3}{\sqrt{3} \times 12.5} = 1.386kA$$

$$I_H = (8.333)(0.1506) = \mathbf{1.255kA}$$

$$I_X = (8.333)1.386 = 11.55kA$$

For a balanced three phase fault, regardless of transformer winding connection, the current as seen from the high side is the same.

03 PROBLEMS SOLVED

Question 9:

The ratings of a three-phase three winding transformer are:

Primary (P): Y connected, 66kV, 20MVA

Secondary (S): Y connected 13.2kV, 15MVA

Tertiary (T): Δ connected, 2.3kV, 5MVA

Neglecting windings resistances and exciting current, the per-unit leakage reactances are:

X_{PS} = 0.08 on a 20MVA, 66kV base

X_{PT} = 0.10 on a 20MVA, 66kV base

X_{ST} = 0.09 on a 15MVA, 13.2kV base

(a) Determine the per-unit reactances X_P, X_S, X_T of the equivalent circuit on a 20MVA, 66kV base at the primary terminals

(b) Purely resistive loads of a 12MW at 13.2kV and 5MW at 2.3kV are connected to the secondary and tertiary sides of the transformer, respectively. Draw the per-unit impedance diagram, showing the per-unit impedances on a 20MVA, 66kV base at the primary terminals

Solution:

(a) $X_{PS} = 0.08$ pu

$X_{PT} = 0.10$ pu

$X_{ST} = 0.09\left(\frac{20}{15}\right)\left(\frac{13.2}{13.2}\right)^2 = 0.12$ pu (using equation 1)

$X_P = \frac{1}{2}(0.08 + 0.10 - 0.12) = \mathbf{0.03}$ **pu** (using equation 8)

$X_S = \frac{1}{2}(0.08 + 0.12 - 0.10) = \mathbf{0.05}$ **pu**

$X_T = \frac{1}{2}(0.10 + 0.12 - 0.08) = \mathbf{0.07}$ **pu**

(b) For single phase load, $P_{1\emptyset} = V \times I$ where $I = \frac{V}{R}\ Amps$

Or, $R = \frac{V^2}{P_{1\emptyset}}\ \Omega$

$R_S = \frac{(13.2)^2}{12} = 14.52\Omega$

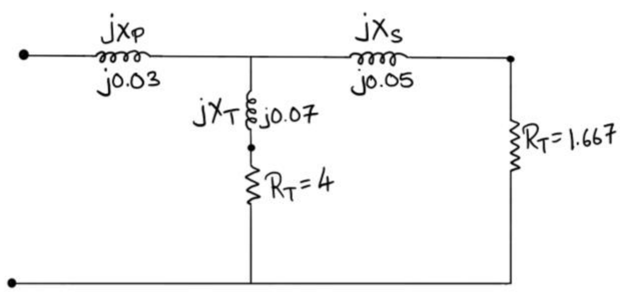
$R_T = \frac{(2.3)^2}{5} = 1.058\Omega$

$R_{S\_base} = \frac{(13.2)^2}{20} = 8.712\Omega$

$R_{T\_base} = \frac{(2.3)^2}{20} = 0.2645\Omega$

$R_{S\_PU} = \frac{14.52}{8.712} = \mathbf{1.667\ pu}$

$R_{T\_PU} = \frac{1.058}{0.2645} = \mathbf{4.0\ pu}$ (using equation 4)



03 PROBLEMS SOLVED

Question 10:

The ratings of a three-phase, three-winding transformer are:

Primary (P): Y connected, 66kV, 15MVA

Secondary (S): Y connected 13.2kV, 10MVA

Tertiary (T): Δ connected, 2.3kV, 5MVA

Neglecting resistances and exciting current, the leakage reactances are:

X_{PS} = 0.07 on a 15MVA, 66kV base

X_{PT} = 0.09 on a 15MVA, 66kV base

X_{ST} = 0.08 on a 10MVA, 13.2kV base

(a) Determine the per-unit reactances of the per-phase equivalent circuit using a base of 15MVA, 66kV for the primary.

(b) An infinite bus, which is a constant voltage source, is connected to the primary of this transformer. A 7.5MVA, 13.2kV synchronous motor with subtransient reactance of 0.2 pu is connected to the transformer secondary. A 5MW 2.3kV three phase resistive load is connected to the tertiary. Choosing a base of 66kV and 15MVA in the primary, draw the impedance diagram of the system in per unit. Neglect transformer exciting current, phase shifts, and all resistances except resistive load.

Solution:

(a) In the primary circuit S_{base} = 15MVA and V_{base} = 66kV

In the secondary circuit S_{base} = 15MVA and V_{base} = 13.2kV

In the tertiary circuit S_{base} = 15MVA and V_{base} = 2.3kV.

Because X_{PS} and X_{PT} are derived using correct S and V base there is no need to change them. However, X_{ST} is on a 10MVA base and is therefore modified to the new base as follows:

$$X_{ST} = 0.08 \times \frac{15}{10} = 0.12\ \Omega$$

With the bases specified, the per-unit reactances of the per-phase equivalent circuit are given by

$$X_P = \frac{1}{2}\left(j0.07 + j0.09 - j0.12\right) = \boldsymbol{j0.02} \text{ pu}$$

$$X_S = \frac{1}{2}\,(j0.07 + j0.12 - j0.09) = \boldsymbol{j0.05} \text{ pu}$$

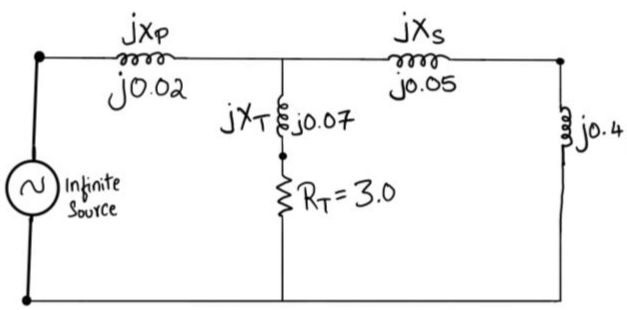
$$X_T = \frac{1}{2}\,(j0.09 + j0.12 - j0.07) = \boldsymbol{j0.07} \text{ pu}$$

(b) On a base of 5MVA 2.3kV in the tertiary, the resistance is 1.0pu.

On 15MVA 2.3kV, this resistance using equation 1 is $\mathbf{1} \times \frac{15}{5} = \mathbf{3}$ pu.

For the motor, on a base of 15MVA 13.2kV the per unit reactance is,

$X_d^{"} = 0.2 \times \frac{15}{7.5} = \mathbf{0.4}$ pu



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