

POWER SYSTEMS ENGINEERING

PER UNIT SYSTEMS

**TEN TOTAL PROBLEMS SOLVED WITH
DETAILED EXPLANATION**

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FROM THE AUTHOR

I CAME ACROSS PER UNIT SYSTEMS WHEN I WAS STUDYING EE FOR MY BACHELORS. I CAME ACROSS IT AGAIN DURING MY MASTERS PROGRAM. AND NOW TEN YEARS LATER - DECEMBER 2018, I STILL DEAL WITH IT AS A SUBSTATION ELECTRICAL ENGINEER AT A LOCAL ELECTRIC UTILITY.

DO NOT TAKE THIS TOPIC LIGHTLY IF YOU PLAN ON FOLLOWING MY PATH INTO THE POWER INDUSTRY.

FOR MORE INFORMATION ON THIS TOPIC AND OTHERS PERTAINING TO DESIGN OF POWER GRID, PLEASE VISIT MY WEB RESOURCE:

[HTTPS://PEGURU.COM](https://peguru.com)

ALL THE BEST FOR YOUR FUTURE ENDEAVORS.

A handwritten signature in black ink, appearing to read 'Aleen', with a horizontal line underneath the name.

ALEEN

01 WHY USE PER UNIT SYSTEMS

Fundamental to any power system analysis is the know-how of per unit systems. This metric is widely used to describe voltages, currents, and impedances in a power system. This book, supplemented by plenty of examples, will explain how to calculate these parameters for any component anywhere in the power system. Let's begin with why use per unit systems.

1. Transformers and per unit systems

The reason the per unit system is employed is because it simplifies calculation of voltages, currents, and impedances of a power system network. Imagine a 3-phase system with a generator, a transformer, a T-line, another transformer, and a load. Every time you cross the transformers, the currents and voltages change. The impedance of the transformer referred to the primary side is different from the impedance of the same transformer referred to the secondary side. It is imperative to use the appropriate impedance based on which side of the transformer you are on, which if not accounted for, introduces serious calculation errors.

Using actual values is fine if not for the transformers. They complicate matters. Generators and motors, like transformers, have different impedance when referred to either stator or rotor winding.

Deriving a per unit impedance values for these components enables easy calculation. This is because the per unit value remains the same whether you are on the primary side or the secondary side of the equipment.

2. The $\sqrt{3}$ factor

The second factor in choosing per unit systems is the avoidance of $\sqrt{3}$ factor in the calculations. If you have ever tried calculating either the sending end voltage or the receiving end voltage in a system, you would know how important it is to factor in the $\sqrt{3}$ multiplier. This is because the voltages are always specified in phase to phase (L-L)

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values while the voltage drop across the impedances are calculated on per phase (L-N) basis. You will have to use $\sqrt{3}$ factor extensively to switch between L-L and L-N values.

When using the per unit system, a 3-phase balanced network is reduced to a 1-phase equivalent where the $\sqrt{3}$ isn't even in the picture.

3. Intuition enabler

The third factor is the meaning expressed by the per unit values. Look at the figure below.

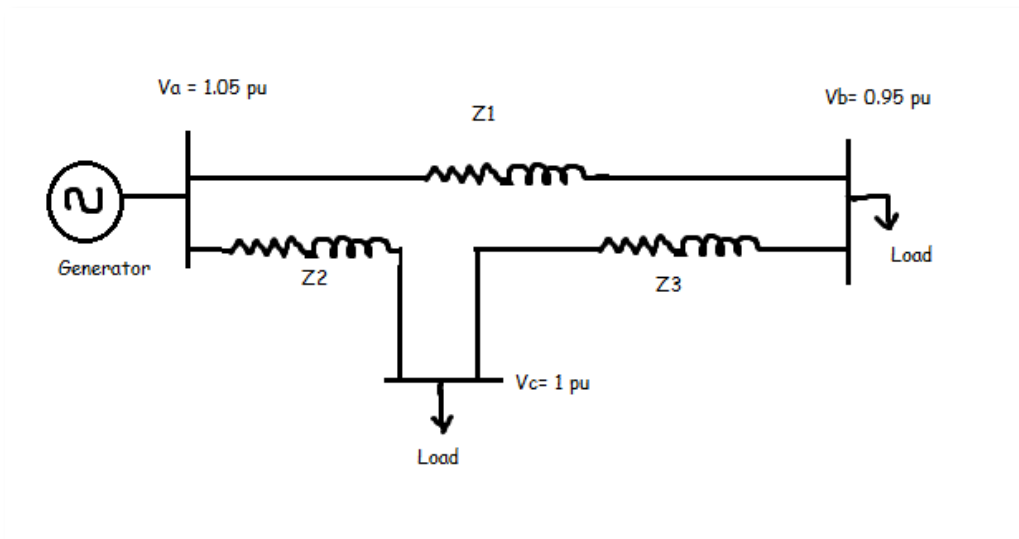


Figure 1: Voltage Per Unit

What does it tell about the system voltage profile? The generator is generating 5% above rated voltage. The load at bus C has no drop in voltage (1 pu) while the load at bus B sees 5% drop in voltage. Essentially, the voltage drops as you move away from the generator and closer to the load. It is easier to visualize this using per units than using actual values since the per unit values are normalized.

01 WHY USE PER UNIT SYSTEMS

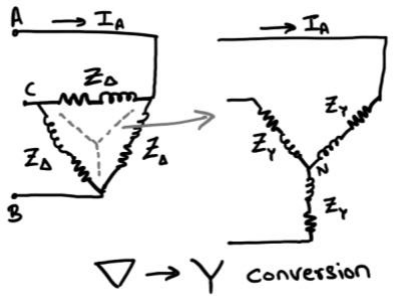
Also, someone with experience knows what the per unit values, especially impedance, of certain transformers or other devices in the system could be. It is easier for them to analyze the power network by making reasonable assumptions.

Other factors for employing per unit system include

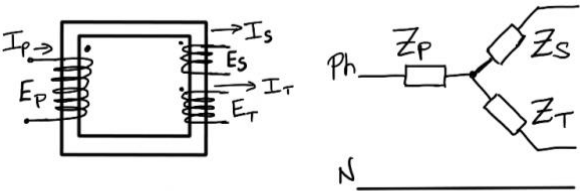
- enables the use of symmetrical components.
- widely used by manufacturers to specify their equipment – making it easy for direct analysis.

At this point it is sufficing to say the per unit system is indispensable in analyzing the power system

02 EQUATIONS

No.	Equation	Comments
1	$Z_{pu_{new}} = Z_{pu_{old}} \times \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{kV_{old}}{kV_{new}} \right)^2$	Normally per unit impedance of equipment is furnished on its nameplate S and V rating. Use this equation to calculate new impedance when either one or both ratings need to change to another base.
2	$Z_{pu} = Z_{\Omega} \times \frac{MVA_{base}}{kV_{base}^2}$ <p>Where, $Z_{base} = \frac{kV_{base}^2}{MVA_{base}}$</p>	Useful in calculating transmission line per unit impedance. Plug in line impedance in ohms and the desired S_{base} and V_{base} .
3	$V_{Secbase} = V_{Pribase} \times \frac{N_s}{N_p}$ <p>Since voltage developed in transformer is proportional to turns ratio</p> $V_{Secbase} = V_{Pribase} \times \frac{V_s}{V_p}$	Calculate the new V_{base} on the secondary side of transformer provided a V_{base} is selected on the primary side or vice versa. See solved problems 1 or 2.
4	$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$	Formula for per unit quantity. The same applies for calculating V, I, or S in per unit.
5	$Z_{pri_referred} = \left(\frac{N_1}{N_2} \right)^2 \times Z_{Sec_impedance}$	Use this formula to transfer the impedance on the secondary side of transformer to the primary side.
6	 $Z_Y = \frac{Z_{\Delta}}{3}$	<p>A balanced Δ load can be converted to a balanced Y load using this equation. Also, the line current entering Y will be equal to that entering delta. The phase angles are not affected and remain the same.</p> <p>A balanced Y can be converted into Δ using this same expression.</p> <p>See solved problem 5.</p>
7	$V_{AB} = \sqrt{3}V_{an} \angle + 30^\circ$ $V_{BC} = \sqrt{3}V_{bn} \angle + 30^\circ$	In a balanced Y connected load, line-to-line voltages are $\sqrt{3}$ times line-to-neutral and lead by 30° .

02 EQUATIONS

	$V_{CA} = \sqrt{3}V_{cn}\angle + 30^\circ$ <p style="text-align: center;">complex power:</p> $S_{3\phi} = \sqrt{3}V_{LL}I_L^* = P_{3\phi} + jQ_{3\phi} = \sqrt{3}V_{LL}I_L \cos \phi + j\sqrt{3}V_{LL}I_L \sin \phi$	<p>In a balanced Δ connected load, line currents into the load are $\sqrt{3}$ times the Δ-load currents and lag by 30°.</p> <p>ϕ is the angle between voltage and current.</p> <p>In purely resistive network, current is in phase with voltage. Therefore $\phi=0$. In inductive network, current lags voltage. In capacitive network, current leads voltage.</p> <p>See solved problem 4.</p>
8	<p style="text-align: center;">Equation to calculate impedance of a single phase three-winding transformer</p> <div style="text-align: center;">  </div> <p style="text-align: center;">Single Phase Three Winding</p> $Z_P = \frac{1}{2}(Z_{PS} + Z_{PT} - Z_{ST})$ $Z_S = \frac{1}{2}(Z_{PS} + Z_{ST} - Z_{PT})$ $Z_T = \frac{1}{2}(Z_{PT} + Z_{ST} - Z_{PS})$	<p>Here,</p> <p>Z_{PS}= per unit leakage impedance measured from primary winding, with secondary shorted and tertiary open</p> <p>Z_{PT}= per unit leakage impedance measured from primary winding, with tertiary shorted and secondary open</p> <p>Z_{ST}= per unit leakage impedance measured from secondary winding, with tertiary shorted and primary open</p> <p>These impedances (Z_{PS}, Z_{PT} & Z_{ST}) can be obtained from the transformer manufacturer based on their short-circuit tests pursuant to IEEE standards.</p>

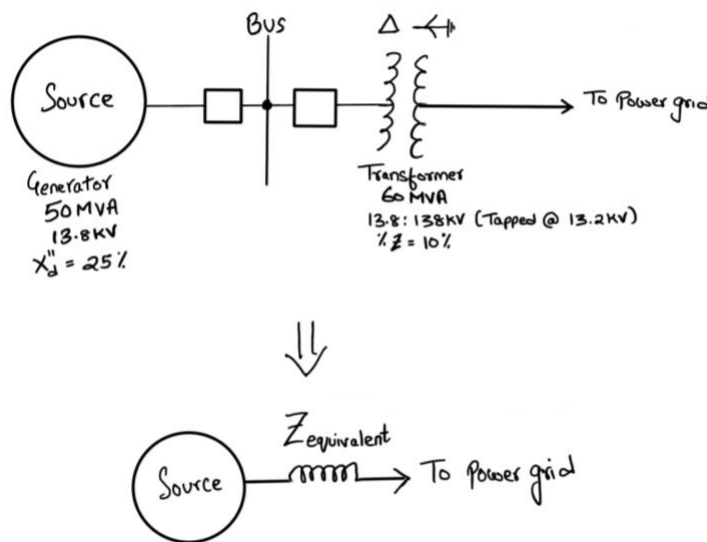
03 PROBLEMS SOLVED

For the problems solved below, few general assumptions need to be made about the power system. These may be reiterated in the question.

1. Line shunt capacitance of a transmission line is neglected.
2. Magnetizing reactance of transformer is ignored. Only leakage reactance is used in the per unit impedance drawings.

Question 1:

A generator and its step-up transformer are shown. The (direct-axis) sub-transient reactance of generator (in per unit) and transformer impedance (in per unit) are provided. Both per unit impedances are based on their nameplate rating. Calculate equivalent impedance of this system using $S_{base} = 100\text{MVA}$ and $V_{base} = 132\text{kV}$.



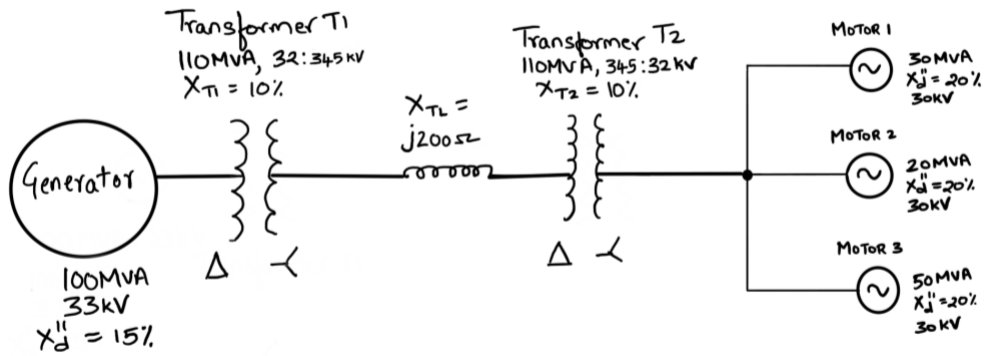
Solution:

[DONATE AND DOWNLOAD THE FULL VERSION TO VIEW THE SOLUTION. THANK YOU.]

03 PROBLEMS SOLVED

Question 2:

For the power system shown below, draw the per unit impedance circuit. Select the generator S and V ratings as the base quantities.



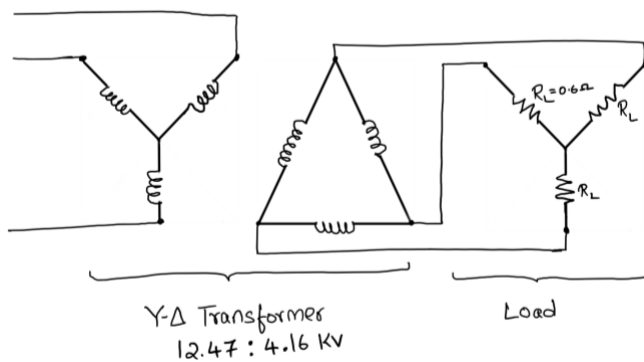
Solution:

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03 PROBLEMS SOLVED

Question 3:

Three single phase, two-winding transformers, each rated 25 MVA, 7.2/4.16kV, are connected to form a three-phase Y- Δ bank with a balanced Y connected resistive load of 0.6Ω per phase on the low voltage side. By choosing a base of 75MVA (three phase) and 12.47kV(line-to-line) for the high voltage side of the transformer bank, specify the base quantities for the low-voltage side. Then determine the load resistance of R_L in ohms referred to the high-voltage side and the per-unit value of this load resistance on the chosen base.



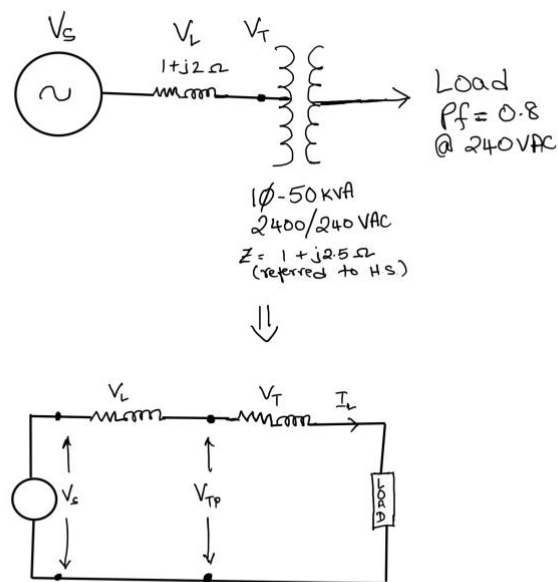
Solution:

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03 PROBLEMS SOLVED

Question 4:

A single-phase 50kVA, 2400/240volt, 60Hz distribution transformer is used as a step-down transformer at the load end of a 2400-volt feeder whose series impedance is $(1.0+j2.0)$ ohms. The equivalent series impedance of the transformer $(1.0 +j2.5)$ ohms referred to the high-voltage (primary) side. The transformer is delivering 1 per unit current at 0.8 power factor lagging and at rated secondary voltage. Neglecting the transformer exciting current, determine (a) the voltage at the transformer primary terminals, (b) the voltage at the sending end of the feeder, and (c) the real and reactive power delivered to the sending end of the feeder. (d) Using the transformer ratings as base quantities, work the problem in per-unit.



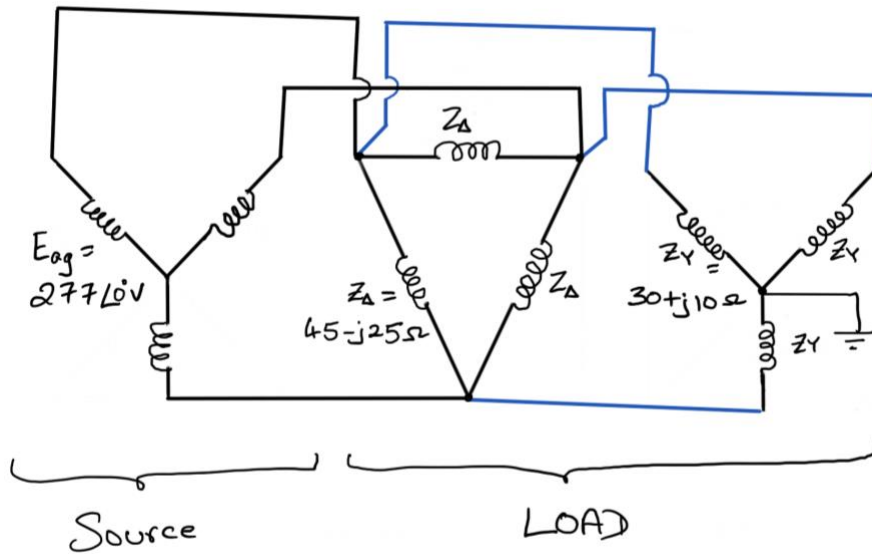
Solution:

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03 PROBLEMS SOLVED

Question 5:

A balanced Y-connected voltage source with $E_{ag} = 277\angle 0^\circ$ volts is applied to a balanced Y-load in parallel with a balanced- Δ load, where $Z_Y = 30 + j10$ and $Z_\Delta = 45 - j25$ ohms. The Y load is solidly grounded. Using base values of $S_{base1\phi} = 5kVA$ and $V_{baseLN} = 277$ volts, calculate the source current I_a in per-unit and in amperes.



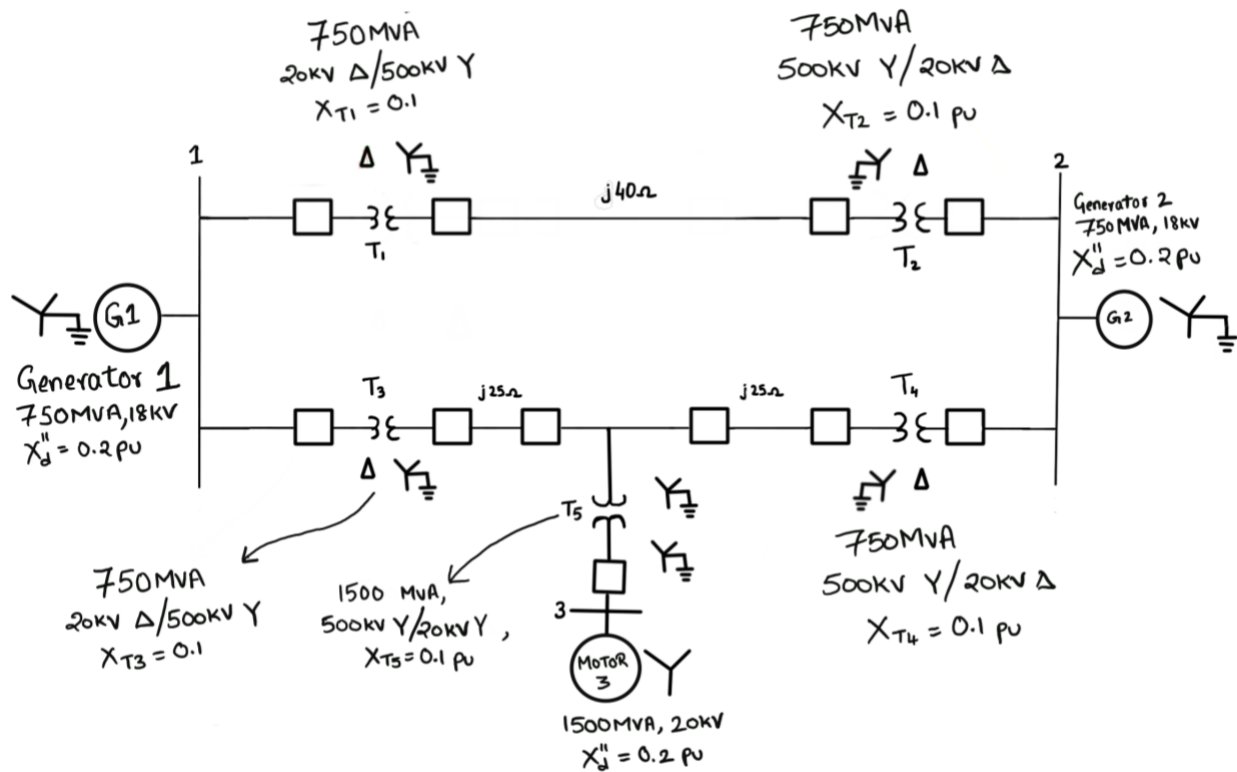
Solution:

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03 PROBLEMS SOLVED

Question 6:

Consider the single-line diagram of the power system shown in figure.



Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the equivalent reactance diagram. Use a base of 100 MVA and 500 kV for the 40-ohm line. Determine the per-unit reactances.

Solution:

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Question 7:

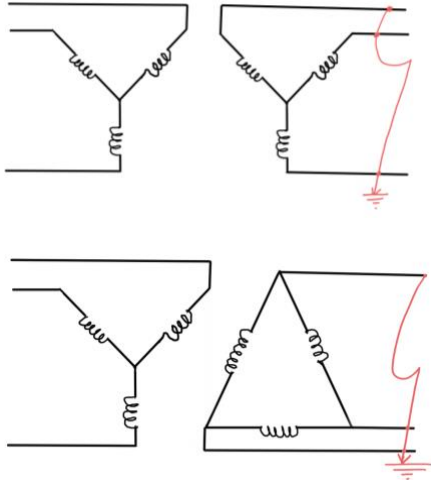
For the power system problem above, the synchronous motor absorbs 1200 MW at 0.8 power factor leading with the bus 3 voltage at 18 kV. Determine the bus 1 and bus 2 voltages in kV. Assume that generators 1 and 2 deliver equal real powers and equal reactive powers. Also assume a balanced three-phase system with positive-sequence sources.

Solution: [DONATE AND DOWNLOAD THE FULL VERSION TO VIEW THE SOLUTION. THANK YOU.]

03 PROBLEMS SOLVED

Question 8:

Three single phase transformers, each rated at 10MVA 66.4/12.5kV, 60Hz, with an equivalent series reactance of 0.12 per unit divided equally between primary and secondary, are connected in a three-phase bank. The high-voltage windings are Y connected and their terminals are directly connected to a 115-kV three-phase bus. The secondary terminals are all shorted together. Find the currents entering the high-voltage terminals if the low-voltage windings are (a) Y connected, (b) Δ connected.



Solution:

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03 PROBLEMS SOLVED

Question 9:

The ratings of a three-phase three winding transformer are:

Primary (P): Y connected, 66kV, 20MVA

Secondary (S): Y connected 13.2kV, 15MVA

Tertiary (T): Δ connected, 2.3kV, 5MVA

Neglecting windings resistances and exciting current, the per-unit leakage reactances are:

$X_{PS} = 0.08$ on a 20MVA, 66kV base

$X_{PT} = 0.10$ on a 20MVA, 66kV base

$X_{ST} = 0.09$ on a 15MVA, 13.2kV base

- (a) Determine the per-unit reactances X_P, X_S, X_T of the equivalent circuit on a 20MVA, 66kV base at the primary terminals
- (b) Purely resistive loads of a 12MW at 13.2kV and 5MW at 2.3kV are connected to the secondary and tertiary sides of the transformer, respectively. Draw the per-unit impedance diagram, showing the per-unit impedances on a 20MVA, 66kV base at the primary terminals

Solution:

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03 PROBLEMS SOLVED

Question 10:

The ratings of a three-phase, three-winding transformer are:

Primary (P): Y connected, 66kV, 15MVA

Secondary (S): Y connected 13.2kV, 10MVA

Tertiary (T): Δ connected, 2.3kV, 5MVA

Neglecting resistances and exciting current, the leakage reactances are:

$X_{PS} = 0.07$ on a 15MVA, 66kV base

$X_{PT} = 0.09$ on a 15MVA, 66kV base

$X_{ST} = 0.08$ on a 10MVA, 13.2kV base

- (a) Determine the per-unit reactances of the per-phase equivalent circuit using a base of 15MVA, 66kV for the primary.
- (b) An infinite bus, which is a constant voltage source, is connected to the primary of this transformer. A 7.5MVA, 13.2kV synchronous motor with subtransient reactance of 0.2 pu is connected to the transformer secondary. A 5MW 2.3kV three phase resistive load is connected to the tertiary. Choosing a base of 66kV and 15MVA in the primary, draw the impedance diagram of the system in per unit. Neglect transformer exciting current, phase shifts, and all resistances except resistive load.

Solution:

[DONATE AND DOWNLOAD THE FULL VERSION TO VIEW THE SOLUTION. THANK YOU.]

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