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PER UNIT SYSTEMS

The Basics

Per Unit System – An Introduction

Fundamental to any power system analysis is the know-how of per unit systems. This metric is widely used to describe voltages, currents, and impedances in a power system. This article, supplemented by an example, will explain step by step how to calculate these parameters for any component anywhere in the power system. Let's begin with the purpose.

Purpose

1. Transformers and per unit systems

The reason the per unit system is employed is because it simplifies calculation of currents. Imagine a 3-phase system with a generator, a transformer, a T-line, another transformer, and a load. Every time you cross the transformers, the currents and voltages change. The impedance of the transformer referred to the primary side is different from the impedance of the same transformer referred to the secondary side. It is imperitive to use the appropriate impedance based on which side of the transformer you are on, which if not accounted for, introduces serious calculation errors.

Using actual values are fine if not for the transformers. They complicate matters. Generators and motors, like transformers, have different impedance when referred to either stator or rotor winding.

Deriving a per unit impedance values for these components enables easy calculation. This is because the per unit value remains the same whether you are on the primary side or the secondary side of the equipment.

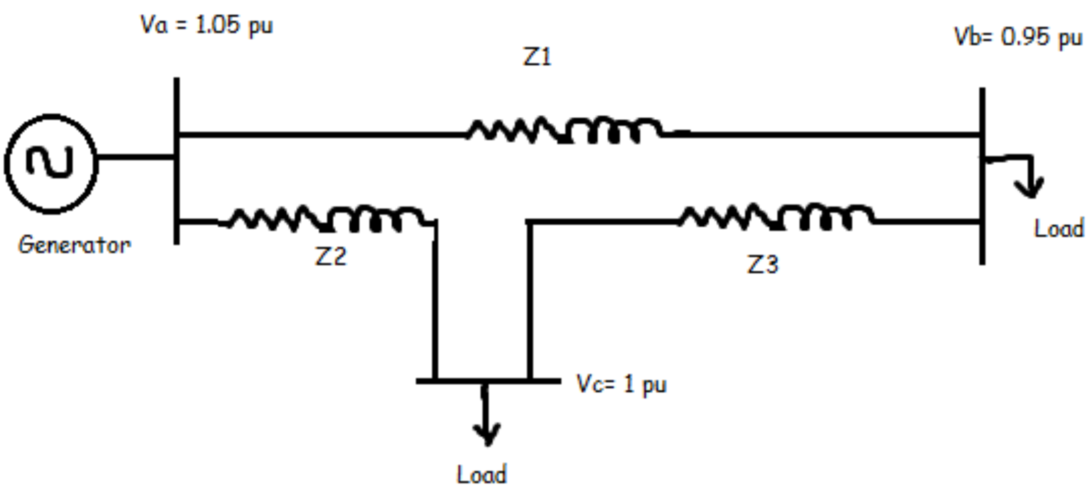
2. The $\sqrt{3}$ factor

The second factor in choosing per unit systems is the avoidance of $\sqrt{3}$ factor in the calculations. If you have ever tried calculating either the sending end voltage or the receiving end voltage in a system, you would know how important it is to factor in the $\sqrt{3}$ multiplier. This is because the voltages are always specified in phase to phase (L-L) values while the voltage drop across the impedances are calculated on per phase (L-N) basis. You will have to use $\sqrt{3}$factor extensively to switch between L-L and L-N values.

When using the per unit system, a 3-phase balanced network is reduced to a 1-phase equivalent where the $\sqrt{3}$ isn't even in the picture. So have fun calculating stuff.

3. Intuition enabler

The third factor is the meaning expressed by the per unit values. Look at the figure below.



What does it tell about the system voltage profile? The generator is generating 5% above rated voltage. The load at bus C has no drop in voltage (1 pu) while the load at bus B sees 5% drop in voltage. Essentially, the voltage drops as you move away from the generator and closer to the load. It is easier to visualize this using per units than using actual values since the per unit values are normalized.

Also, someone with experience knows what the per unit values, especially impedance, of certain transformers or other devices in the system could be. It is easier for him or her to analyze the system by making reasonable assumptions.

Other factors for employing per unit system include

- enables the use of symmetrical components.
- widely used by manufacturers to specify their equipment – making it easy for direct analysis.

At this point it is sufficing to say the per unit system is indispensable in analyzing the power system.

What is a per unit value?

The per unit of any quantity, be it voltage, current, or impedance is given by the following equation:

$$PerUnit_{quantity} = \frac{ActualValue_{quantity}}{BaseValue_{quantity}}$$

Pretty straight forward equation. You have your actual value that you are trying to convert and a base value that is calculated from the given quantities like apparant power (S) and system voltage (V).

Per Unit System – Practice Problem Solved for Easy Understanding

Let's understand the concept of per unit system by solving an example. In the one-line diagram below, the impedance of various components in a power system, typically derived from their nameplates, are presented. The task now is to normalize these values using a common base.

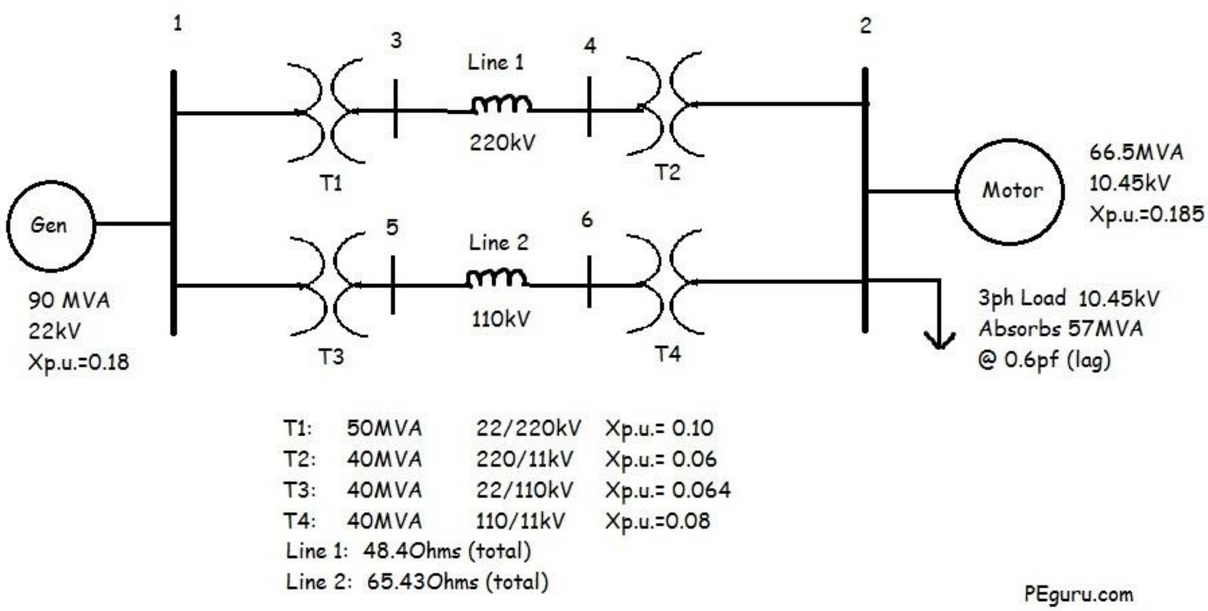


Figure 1: Oneline Diagram of a Power System

Now that you have carefully examined the system and its parameters, the equivalent impedance diagram for the above system would look something like the following.

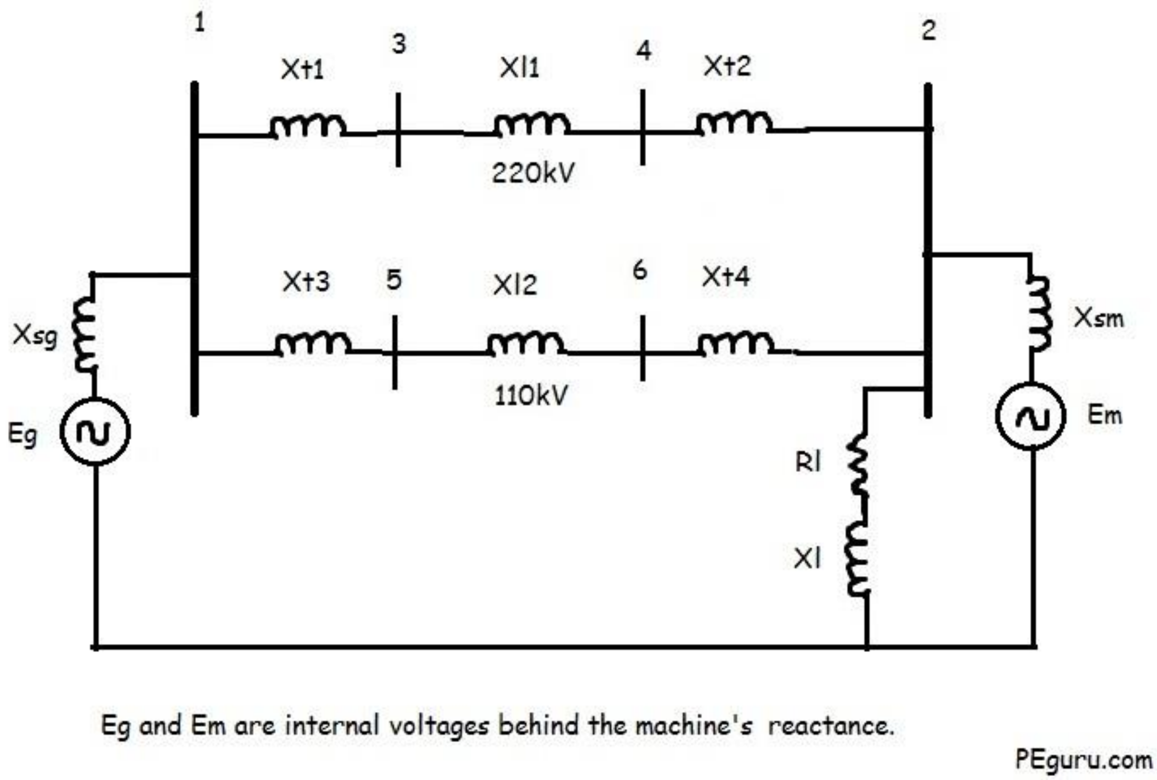


Figure 2: Impedance Diagram of a Power System

Resistive impedance for most components have been ignored. Rotating machines have been replaced with a voltage source behind their internal reactance. Capacitive effects between lines and to ground are ignored as well.

To obtain the new normalized per unit impedances, first we need to figure out the base values (Sbase, Vbase, Zbase) in the power system. Following steps will lead you through the process.

Step 1: Assume a system base

Assume a system wide S_{base} of 100MVA. This is a random assumption and chosen to make calculations easy when calculating the per unit impedances.

So, S_{base} = 100MVA

Step 2: Identify the voltage base

Voltage base in the system is determined by the transformer. For example, with a 22/220kV voltage rating of T1 transformer, the V_{base} on the primary side of T1 is 22kV while the secondary side is 220kV. It does not matter what the voltage rating of the other components are that are encompassed by the V_{base} zone.

See figure below for the voltage bases in the system.

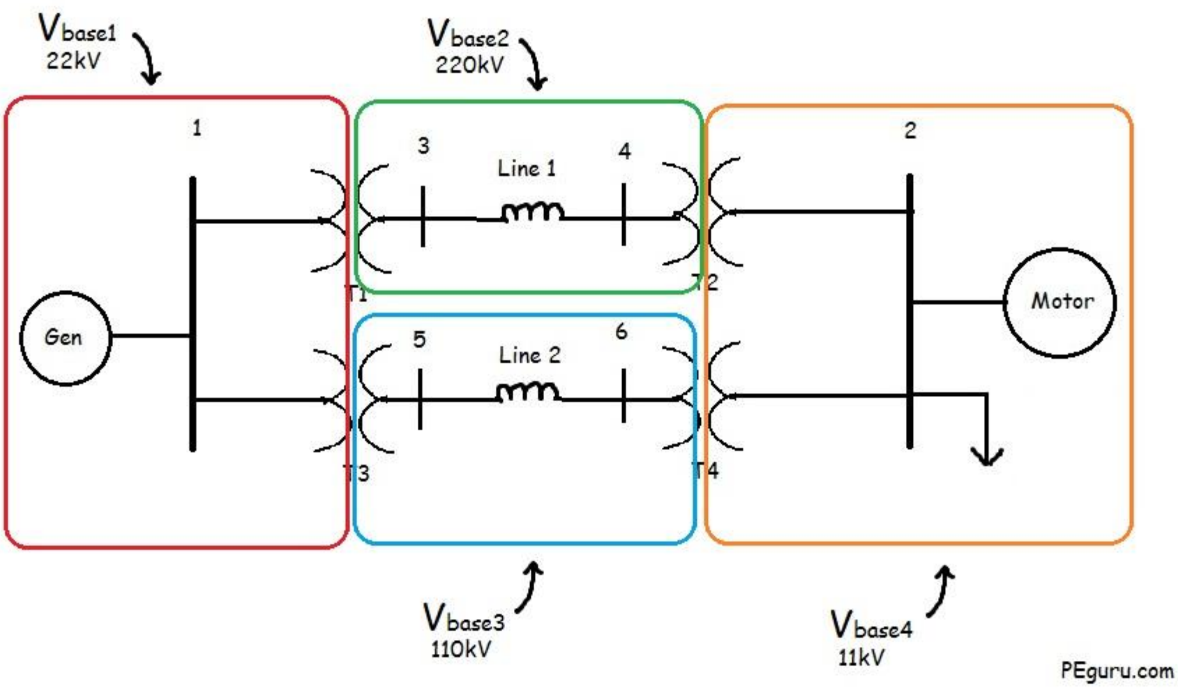


Figure 3: Voltage Base in The Power System

Step 3: Calculate the base impedance

The base impedance is calculated using the following formula:

$$Z_{base} = \frac{kV_{base}^{\ 2}}{S_{baseMVA}} \text{ Ohms} \qquad (1)$$

For T-Line 1: $Z_{base} = \frac{(220)^2}{100}$ = 484 Ohms

For T-Line 2: $Z_{base} = \frac{(110)^2}{100}$ = 121 Ohms

For 3-phase load: $Z_{base} = \frac{(11)^2}{100}$ = 1.21 Ohms

Step 4: Calculate the per unit impedance

The per unit impedance is calculated using the following formulas:

$$Z_{p.u.} = \frac{Z_{actual}}{Z_{base}} \tag{2}$$

$$Z_{p.u._{new}} = Z_{p.u._{old}}\left(\frac{S_{base_{new}}}{S_{base_{old}}}\right)\left(\frac{V_{base_{old}}}{V_{base_{new}}}\right)^2 \tag{3}$$

The voltage ratio in equation (3) is not equivalent to transformers voltage ratio. It is the ratio of the transformer's voltage rating on the primary or secondary side to the system nominal voltage on the same side.

For T-line 1 using equation (2): $X_{l1_{p.u.}} = \frac{48.4}{484}$ = 0.1 pu

For T-line 2 using equation (2): $X_{l2_{p.u.}} = \frac{65.43}{121}$ = 0.5 pu

For 3-Phase load:

Power Factor: $\cos^{-1}(0.6) = \angle 53.13$

Thus, $S_{3\phi}(load) = 57\angle 53.13$

= 1.1495+j1.53267 Ohms

Per unit impedance of 3-phase load using equation (2)= $\frac{1.1495+j1.5326}{1.21}$ = 0.95+j1.2667 pu

For generator, the new per unit reactance using equation (3)

$X_{sg} = 0.18(\frac{100}{90})(\frac{22}{22})^2$ = 0.2 pu

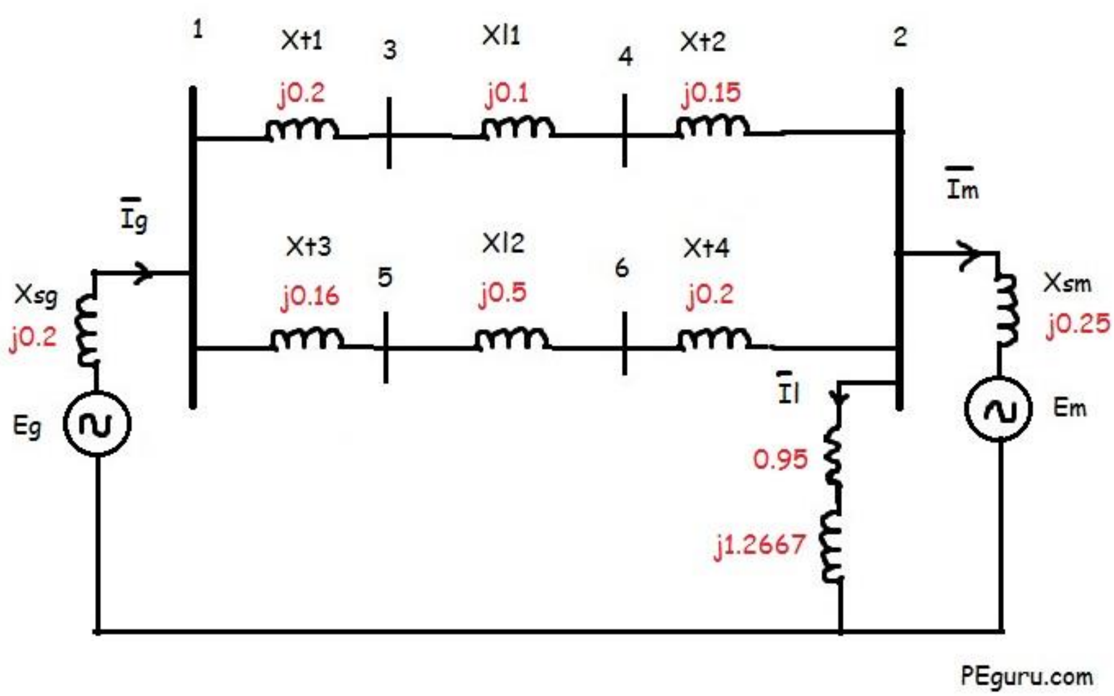
For transformer T1: $X_{t1} = 0.1(\frac{100}{50})(\frac{22}{22})^2$ = 0.2 pu

For transformer T2: $X_{t2} = 0.06(\frac{100}{40})(\frac{220}{220})^2$ = 0.15 pu

For transformer T3: $X_{t3} = 0.064(\frac{100}{40})(\frac{22}{22})^2$ = 0.16 pu

For transformer T4: $X_{t4} = 0.08(\frac{100}{40})(\frac{110}{110})^2$ = 0.2 pu

For Motor, $X_{sm} = 0.185(\frac{100}{66.5})(\frac{10.45}{11})^2$ = 0.25 pu



Summary

1. Assume a Sbase for the entire system.
2. The Vbase is defined by the transformer and any off-nominal tap setting it may have.
3. Zbase is derived from the Sbase and Vbase.
4. The new per unit impedance is obtained by converting the old per unit impedance on old base values to new ones. See equations (2) and (3).

Per Unit System: Problem Solved for Easy Understanding (continued)

In the previous post, we calculated the per unit impedance of each equipment in the power system. In this post we will calculate the full load amps at Bus 2.

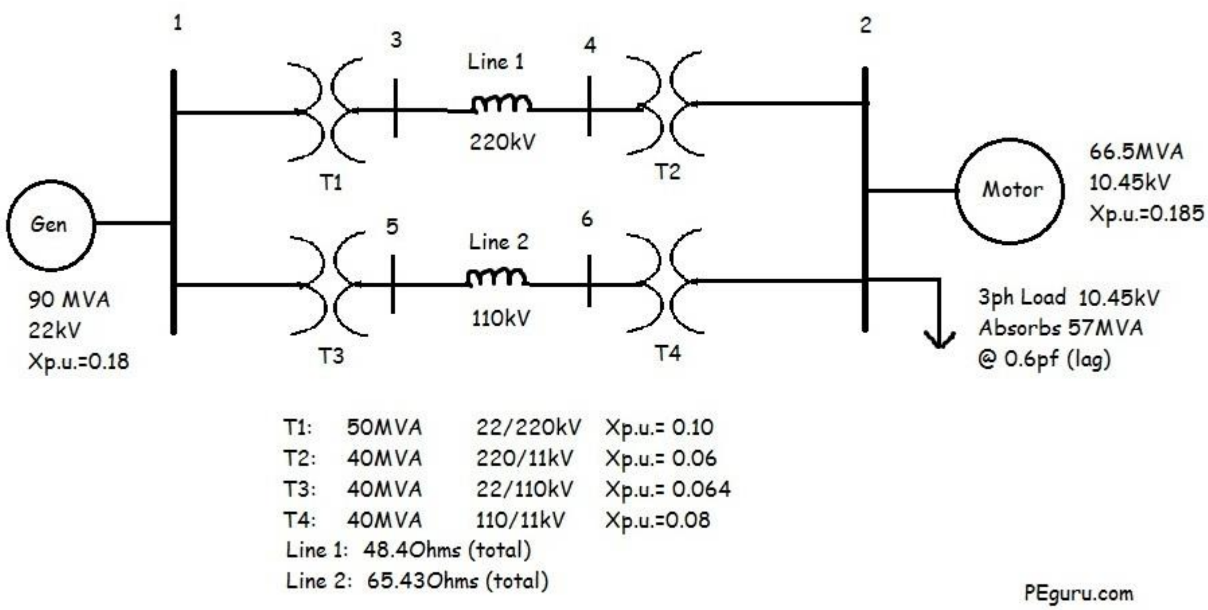


Figure 1: Oneline Diagram of a Power System

In the figure shown, Bus 2 is supplying the power to motor load and another inductive load. The available ratings of the load are also shown.

We need to assume the following before we get too far along

- Bus 2 is the reference bus. That is the phase angle will be 0.
- The motor has a leading power factor of 0.8.

Step 1: Calculate bus voltage

The per unit voltage at Bus 2 is
$V_2 = \frac{10.45}{11}\angle 0 = 0.95\angle 0$

Step 2: Calculate current drawn by motor

The apparent power consumed by motor is
$\overline{S_m} = \frac{66.5}{100}\angle{-36.87}$

The current drawn by the motor is

$$\overline{I_m} = \frac{\overline{S_m^*}}{\overline{V_2}} = \frac{0.665\angle 36.87}{0.95\angle 0} = 0.56 + j0.42$$

Step 3: Calculate current drawn by inductive load

Current drawn by load is

$$\overline{I_L} = \frac{\overline{V_2}}{\overline{Z_L}} = \frac{0.95\angle 0}{0.95+j1.2667} = 0.36 - j0.48$$

Step 4: Calculate total current at Bus 2

Using KCL, the current in each leg equals the current entering at node Bus 2.
I.e. $\overline{I} = \overline{I_m} + \overline{I_L} = 0.92 - j0.06 perunit$

For current in Amperes, calculate

$$I_{act} = I_{pu} \times I_{base} = 8364 - j545$$

$$I_{act} = 8382\angle{-3.7} Amperes$$

8,382Amperes of load current is extremely high on single piece of bus. This happened because we added 66MVA AND 57MVA load on Bus 2. Note that this is not a real-world scenario.